# 8. Probability.

As we will see in Ch. 11, quantum mechanics tells us the observer perceives only one version of reality, in spite of the fact that several versions exist in the wave function. It doesn't tell us which one we will perceive on a given run, but it does tell us the *probability* of perceiving a particular outcome. This introduction of probability into the theory presents a substantial problem for understanding just how the mathematics of quantum mechanics relates to what we perceive. But before we can phrase the problem, we need to understand probability in quantum mechanics in more detail. To illustrate, we will again use the half-silvered mirror experiment of Ch. 6.

**Coefficients.**

We suppose the thickness of the silver on the mirror can be varied, so that different amounts of the light wave function are reflected in the vertical direction or transmitted in the horizontal direction. We will first show how the light wave function changes as we change the thickness of the silver. Before the light hits the mirror, the wave function for a single packet is

$$|\text{before}\rangle = |h\rangle = (1)\ |h\rangle \qquad (1)$$

where the *h* stands for the horizontal part of the path in Fig. (8-1) before the mirror.

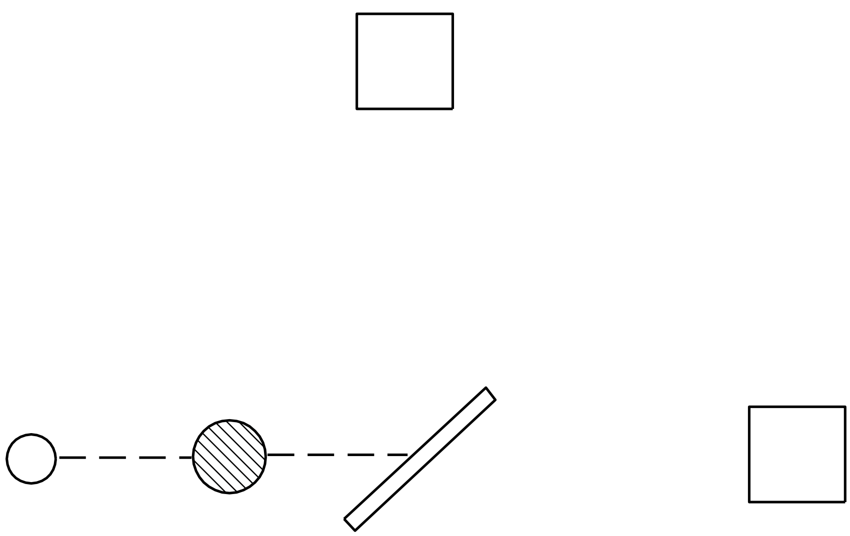

Fig. 8-1

Figure 8-1. Wave function of photon about to hit half-silvered mirror.

If the mirror is truly half silvered, then after it hits the mirror the light wave function is

$$|\text{after}\rangle = (.707)|H\rangle + (.707)|V\rangle. \qquad (2)$$

where $|H\rangle$ stands for the version of the packet on the horizontal path after the mirror, and similarly for $|V\rangle$. But if the mirror is more than half silvered, so that more of the light is reflected on the vertical path, the wave function after the mirror might be

$$|\text{after}\rangle=(.6)|H\rangle+(.8)|V\rangle. \tag{3}$$

where the .6 and .8 correspond to some specific thickness of silver. The diagram corresponding to this state is

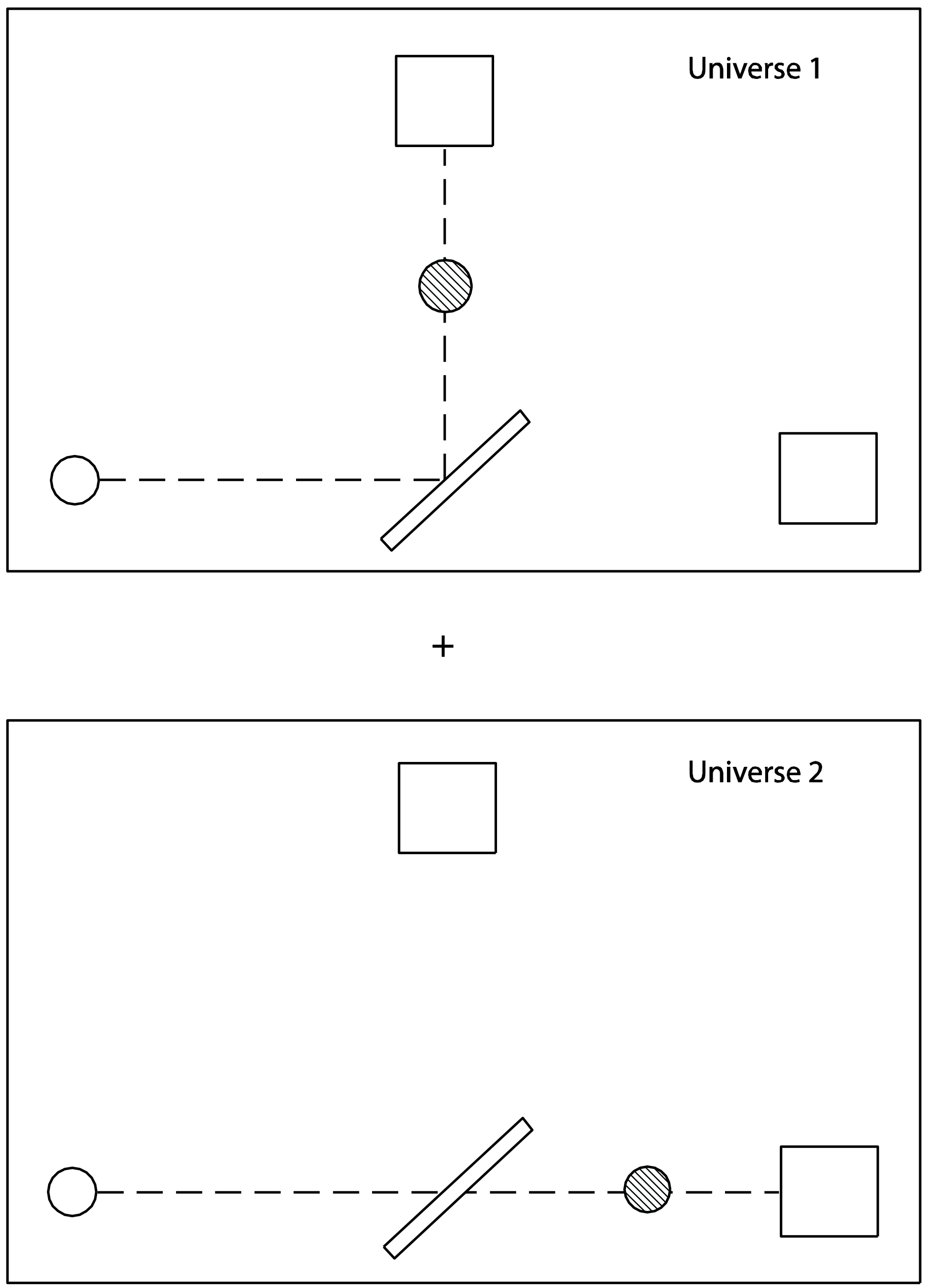


Fig. 8-2

Figure 8-2. Unequal splitting by the mirror. There are two versions of reality, the .8 version (upper diagram) and .6 version (lower diagram).

What is being illustrated in the equations and by the different sizes of the circles in the figures is the idea of the **coefficients**, the numbers in parentheses, in front of the kets—the |h⟩, the |H⟩ and the |V⟩. In Eq. (1) the single coefficient is 1; in Eq. (2) the two coefficients after the mirror are .707 and .707; and in Eq. (3), the two coefficients are .6 and .8.

**Coefficients and the "amount" of the wave function.**
**Conservation of "amount."**

We now go back to the idea from Ch. 4 that the wave function can be visualized as a (water) mist. In this visualization, the 'amount' of the wave function would be the total amount of water in the mist. Because of the way the mathematics of quantum mechanics works (unitarity, A8.1), the *total* amount stays the same for each wave function for all time. This is conservation of amount; the total amount is the same both before and after the mirror in the half-silvered mirror case.

In terms of mathematical formulas, it works this way: The 'amount' for a ket with coefficient 1, say $|V\rangle$, is, by convention, always taken as 1. When there are two (or more) parts to the wave function, as in the RHS of Eq. (3), the total amount in quantum mechanics is **the sum of the squares of the coefficients.** The unitarity property then implies that for any wave function, no matter how it evolves in time, the sum of the squares of the coefficients is always 1 for all time. For example, the (implied) coefficient on the LHS of Eq. (3) is 1, so its square is 1. And then the sum of the squares on the RHS, after the light has hit the mirror is $(.6)^2+(.8)^2$, which is also 1. And the same works for Eq. (2) because $(.707)^2+(.707)^2=1$.

**The probability law.**

It is found **experimentally** that the probability of perceiving a certain state is equal to the coefficient squared. So for example, the probability of perceiving the V detector reading yes in Eq. (8-3) is $(.8)^2=.64$, and the probability of perceiving the H detector reading yes is $(.6)^2=.36$. And the two add up to 1, as they must for probabilities.

**Consistency of the probability law and unitarity.**

From the definition of probability, it is a requirement that the sum of the probabilities must be 1. But because of unitarity, which says the sum of the squares of the coefficients will always be 1, and the fact that probability of state $j$ is equal to $|a(j)|^2$, we are guaranteed that the sum of the probabilities will always be 1. Thus the *experimentally* deduced $|a(j)|^2$ probability law, which implies that the sum of the squares of the coefficients is 1, is consistent with the *theoretical* property of unitarity, which says exactly the same thing!

Further, not only is it consistent, but **if** one assumes there is probability of perception in quantum mechanics, then, using unitarity, one can actually **derive** the coefficient squared probability law. This is done in A8.2. There is, however, currently no way to justify the assumption of probability.

**The problem with probability in quantum mechanics.**

We will discuss this more thoroughly in Ch. 18, but the problem with probability in quantum mechanics is that the basic mathematics of quantum mechanics is *deterministic*; there is no probability at all in the mathematics. Once the wave function starts out, its future is determined forever (just as in classical physics!). And so probability and the coefficient squared probability law must have their origin outside the basic mathematical-conceptual scheme. Explaining probability is the reason—the only reason—why we need an interpretation of quantum mechanics.

**The information quantum mechanics gives us.**

It is important to note that the basic mathematics of quantum mechanics determines the coefficients as well as the possible states of reality. Thus quantum mechanics gives us both **the possible states**, including energy levels, readings on detectors, and so on, and **the probability** of perceiving each state.

**Another form for the probability law.**

The probability law is sometimes stated by saying that $|\psi(x)|^2$ is proportional to the probability of finding a particle at *x*, where $\psi(x)$ is the wave function at *x*. In one sense this is not a good statement of the probability law because there is no evidence for particles. But in another sense, it is essentially just the same as the coefficient squared probability law, with position, *x*, taking the place of the state designator, say, *j* (and "the probability of finding a particle at *x*" replaced by "the probability of perceiving a particle-like disturbance at *x*").

**Evaluation.**

A small minority of physicists claim probability of perception can be deduced from the mathematics of quantum mechanics alone. But it cannot because there is no probability of perceiving one version or the other in the mathematics; all versions are perceived on every run. This will be discussed more fully in Ch. 18. Aside from this point, all physicists would agree with the discussion of probability given here.