12. Particle-Like Properties of Matter.
Mass, energy, momentum, spin, charge.

Over the next five chapters, we will consider the evidence for particles. One might think the evidence is overwhelmingly diverse, but it is not; there are just a few basic observations that are used to support this concept. Our goal is to show these basic observations can all be accounted for by the mathematics of quantum mechanics and the properties of the wave functions. So although we can't actually prove particles don't exist, it is consistent with all observations to assume the world is made solely from wave functions/state vectors.

Only one version of reality is perceived in quantum mechanics.

The first step was taken in Ch. 11. There we showed that only one version of reality can be perceived, exactly *as if* the world was made of objectively existing particles (where "objectively" means there is a single, unique version of the object). So in this way at least, wave functions, with their many versions of reality, imitate a world made of objectively existing particles.

The particle-like properties of mass, energy, momentum, spin, and charge.

The next step is to consider the properties of mass, energy, momentum, spin and charge, which have conventionally been associated with particles since the time of Newton. These properties, along with locality, essentially *define* the concept of particles.

Mass. Mass is related to weight; if something is heavier, it has more mass. But weight depends on gravity, the attractive force between an object and the earth, whereas mass is independent of the gravitational force. A rock has the same mass on the moon as on earth, but not the same weight. Mass might best be classically described as resistance to acceleration; the more mass, the harder it is to accelerate an object.

Charge. Charge has to do with how particles interact. We are most familiar with electrical charge, where like charges repel and unlike attract. But there are also charges associated with the strong nuclear force and with the weak forces that govern neutrinos.

Energy and momentum are attributes of matter that are related to the velocity and mass of the object. Their precise definition, I think, is not important here, except perhaps for one case. If an electron (for example) is just sitting there, without moving, it still has a "rest" energy of $m_e c^2$, Einstein's famous formula (m_e is the mass, c is the speed of light).

Spin. Spin is actually a quantum mechanical term. In classical physics, and also often in quantum mechanics, it is called angular momentum. Classically, if one particle is in orbit around another, the particle has angular momentum, the momentum of circular motion, so to speak. Traditionally, one visualizes a particle with spin as a whirling top. But there is no top, it's just a conceptually easy (but somewhat misleading) way to think of it.

In classical Newtonian physics, mass and charge are thought of as basic properties of matter which belong to and are carried by particles. Energy, momentum, and angular momentum are classically thought of as properties due to the motion of particles.

Our goal here is to show that these five particle-*like* attributes can actually be shown to be **properties of the wave function** (or more technically the state vector). This means there is no reason to assume, on this account at least, that there are actual, objectively existing particles, separate from the wave function, which carry mass, spin, charge and so forth. Or to put it another way, these particle-*like* properties cannot be used as evidence for the existence of particles because they can be explained otherwise; they can be **mathematically shown** to be properties of the wave functions.

Classifying solutions to equations in quantum mechanics.

The mathematical reasoning starts with a property of solutions to equations. In simple algebraic equations, such as 3x+4=10, there is only a single solution (x=2). But almost all the equations in quantum mechanics are calculus equations, which typically have an *infinite number* of solutions. This means that, to avoid chaos, and to make sure the solution fits the relevant physical problem, it is often important to find some way of *classifying* solutions to calculus equations.

The classification process and related reasoning for quantum mechanics, where almost all the equations are calculus equations, is intriguing. We will describe it here as it relates to rotations in three dimensions.

(1). It starts from the **physical observation** that the outcome of experiments on the atomic level does not depend on how the experimental apparatus is oriented. It can be oriented North-South, East-West, up-down, or at some angle between these, and that makes no difference to the outcome of the experiments (provided gravity can be ignored).

(2). This physical observation has a reflection in the **form of the equations**, say the Schrödinger equation for the hydrogen atom. They must have a certain symmetry. The equations "look the same" when one changes the orientation of the experimental apparatus. (See A12.1.)

(3). The form dictated by the symmetry then has a consequence for the **classification** of solutions. In the case where the orientation of the apparatus in three-dimensional space makes no difference, the form of the equations implies solutions can be classified according to their spin (angular momentum).

(4). The classification is not just mathematical; it has **physically measurable consequences**. The electron-like wave function in the hydrogen atom has measureable angular momentum which agrees with the result derived from the mathematics of quantum mechanics.

(5). Finally, the mathematics yields a **conservation law**; in the hydrogen atom example, the actual numerical value of the total angular momentum stays the same no matter what happens *within* the system. And the mathematics leads to **addition rules** when there are several particle-like states present.

Quantization.

There is one more interesting property in the case of spin. The mathematics tells us, and experiment confirms, that in the hydrogen atom case spin can have only integer values, 0, 1, 2, … in the appropriate units. That is—in contrast to the classical case, where angular momentum can take on any value—angular momentum is **quantized**; it can take on only certain discrete values rather than being able to take on a continuum of values!

This is an amazing result. One gets a physically measurable property, a conservation law, and the experimentally confirmed quantization,out of almost nothing, just the fact that the orientation of the apparatus makes no difference!

Spin "belongs to" the wave function.

For the point we are trying to make, it is important to note that in any reasonable sense the quantized spin deduced above can be said to "belong to" or "be carried by" the wave function (state vector). There is no reason within the derivation of the quantized values of spin to suppose it 'belongs to' an actual particulate electron because the only 'object' that enters the mathematics and the reasoning is the wave function.

Classification of particle states from space-time symmetries.

Now we can generalize from our example in which only invariance to rotation in three dimensions was used for illustration. First, experimental results do not depend on *where* the experiment is done, or *when* (translational invariance). Further, Einstein deduced relativity which says that the equations of physics must be invariant under certain relativistic rotations involving both space and time. (They include rotations in three-dimensional space, but they also involve velocity.) These invariances—where, when, four-dimensional relativistic rotations—then have a number of consequences for the form of the equations of quantum mechanics, and for classifying the solutions to the equations. (See A12.2, on group representation theory).

First, the invariance under *where* and *when* implies the solutions—the wave functions— can be labeled by **energy and momentum**. Further the mathematical properties associated with these **labels** correspond exactly to the **physically measured properties** of energy and momentum. In addition, one gets **conservation** of energy and momentum; for an isolated system, the total energy and momentum remain the same forever. And the **addition laws** implied by quantum mechanics for energy and momentum when one has more than one 'particle' are just the usual, experimentally verified laws.

Second, if we also take into account invariance under Einstein's relativistic rotations, then the solutions acquire two additional properties—**spin** (analogous to the angular momentum of the hydrogen atom) and **mass**. In this case, the allowed values for spin are still quantized, but they also include *half*-integer values; 0, 1/2, 1, 3/2, 2,… (The possible values for mass are not limited by the invariances). See A12.3 for the derivation of the quantization.

Like the wave function, there is really no good way of visualizing the spin of an elementary 'particle' in quantum mechanics. When we say an electron has spin ½ , there is no object, or even wave function, that is actually spinning in space.

The spin of an elementary particle has no analog in classical physics or the world of our senses.

This result borders on the miraculous! Mass itself, perhaps the most basic property of matter, is a consequence of the simple observation that the outcomes of experiments don't depend on the orientation (generalized to include relativistic rotations) or position of the apparatus! (See A12.4 on speculation about a theory of gravity based on this idea.) And the mathematically allowed quantized values for the angular momentum—and *only* these values—are exactly observed; electrons, quarks and neutrinos, for example, have spin ½ while the photon has spin 1 (and no particle has, for example, spin 2/5). This provides a great example of Wigner's "unreasonable effectiveness of mathematics." (And in fact, it was Wigner who derived these group theoretic results on mass, spin and so on.)

Internal symmetries and charge.

The only remaining property is charge. The charges have the same invariance-under-rotation type of origin as spin, only in a peculiar way. It was found experimentally that the proton, neutron and all other strongly interacting "elementary" particles were made up of three more elementary particles—quarks. The only way to make sense of all the various experiments was to suppose the quarks came in three 'colors' (nothing to do with actual colors, of course; just colorful language). Further the theory had to be invariant—did not change form— under (complex) rotations in the color space, just as the hydrogen atom theory was invariant under rotations in our ordinary three-dimensional space. And the *labels* associated with this invariance are the strong charges.

A similar argument can also be made for the electromagnetic and weak charges, with all charges quantized as integer multiples of the basic charges. (In the case of quarks, the basic electrical charge is 1/3 the charge on the electron.) Thus, just as mass and so on are labels, and physical properties, associated with the wave function due to invariance under space-time rotations and translations, so the three types of charges—strong, electromagnetic, and weak—are labels, and physical properties, associated with the wave function due to an "internal"—"within the particle," so to speak, not in extended space and time—set of rotations in some abstract space.

So we see that the particle-**like** properties—mass, energy, momentum, spin, and the charges—are found, through the invariance argument, to actually be **properties of the wave functions** (state vectors). Or more carefully, these properties can be attributed to the wave function, so they cannot be used as evidence for the existence of particles.

From now on, we may still use the word particle, but we mean a **particle-like wave function**. The word "electron" refers to a wave function with mass m_e, electrical charge $-e$, and spin ½ while "photon" refers to a wave function with 0 mass, 0 charge and spin 1.

Ket notation.

The 'state' of a particle-like system is given by specifying the values of the various quantities—mass and so on. So the general particle-like state in ket notation (A6.2) is

$$|\psi\rangle = |m, E, p, S, s_z, Q\rangle \tag{1}$$

where *m* is the mass, *E* is the energy, *p* is the momentum, *S* is the total angular momentum, s_z is the *z* component of angular momentum, and *Q* stands for all three charges. The problem is that we will show there is no evidence for particles. So that raises the question of what the ket $|m, E, p, S, s_z, Q\rangle$ stands for. This is the fourth mystery, addressed in Part IV.

Abstractness.

The internal symmetry idea is very abstract, not directly related at all to things we can actually see or touch. How can we trust such an abstract argument when it is so distant from our everyday perceptions?

The answer a physicist would give is that abstractness permeates much of modern physics (quantum mechanics) and so one must learn to live with it. We can't actually see spin, for example, but it works so well in describing so much data from varied experiments that physicists are convinced of the "reality" of spin. The same holds for internal symmetries. They explain so much data from so many varied experiments that they are as real—at least to a physicist—as what we actually see.

A couple of further thoughts on what we can and cannot see, and mathematics. In our attempt to probe what matter is like, physics experiments constitute a vast extension of our unaided senses. So it seems unnecessarily restrictive to say that we can only conceptually work with what we can see with the naked eye.

The theories of physics are the distillation of the results of millions of experiments. They are mathematical-conceptual models of physical reality, models which work in many varied circumstances and never, in the case of quantum mechanics at least, lead to incorrect results. Such concepts in this context are the wave functions and their associated spin, charge, and mass. It seems appropriate to suppose these properties correspond to "real" properties of matter, just as real as the properties we directly perceive, even if they transcend the concepts and sense-perceptions of our everyday world. In the case of internal symmetries, one might say the theory points to dimensions of reality we cannot directly perceive (see Part IV).

A somewhat related thought comes up from a neuroscientific or philosophical point of view. Our everyday **concepts** of the nature of the physical world—which correspond to neural firing patterns—have to do with our lifetime of perceiving the physical world. So those concepts do not have to do with the 'actual' nature of physical reality; they only have to do with its *perceived* nature and the way those perceptions are modeled in the circuitry of the brain. In that sense then, our everyday perceptions of the physical world can never reveal the actual nature of physical reality (see Part IV). But that doesn't mean the actual nature of physical reality is forever hidden. We can deduce the mathematical equations matter obeys from experiment, and we can then hope to deduce the actual nature of matter from the mathematics. This idea has already implicitly come up in Part I; we don't directly see the wave function, but we assume it "exists" because it gives such a good description of what we perceive. This idea will re-emerge even more strongly in Part IV.

Evaluation.

Relatively few physicists are aware of the implications of the symmetry arguments. It shows that the particle-like properties of mass, energy, momentum, spin, and charge, which were thought for centuries to be intrinsic properties of particles, can instead be **mathematically derived** as properties of the wave function. Why is this most important result unknown? Because the mathematical discipline of group representation theory used in the argument is not well-known among physicists. But there is no doubt it is correct. It was originally given by the Nobel laureate E. P. Wigner in 1939 and has never been challenged.

Again, it is absolutely astonishing that invariance of the results of experiments to orientation or position should imply that wave functions have the particle-like properties of mass, energy, momentum, quantized spin, and charge.

Abstractness. It would be interesting to take a poll of physicists and see how many consider the wave function, spin, internal symmetries, and even quarks to be "real," rather than corresponding to a mathematical scheme that the real world happens to follow. My guess is that it would split as follows: about 40% would say the abstract quantities correspond to something real; about 30% would say quantum mechanics is just a mathematical scheme that the real world (for unknown reasons) follows; and about 30% would say the question makes no sense because there's no good way to define or understand "real."