A19.2 The GRW-Pearle Mathematical Collapse Model.

The Model.

The time evolution of this proposed mathematical model of collapse, [1], [2], [3] is governed by the Hamiltonian equation for the state vector $|\Psi, \mathrm{t}\rangle$;

$$\frac{\partial|\Psi,\mathrm{t}\rangle}{\partial \mathrm{t}} = -iH|\Psi,\mathrm{t}\rangle + \left(\sum_n (\eta_n w_n(t) - \lambda\eta_n^2)\right)|\Psi,\mathrm{t}\rangle \tag{1}$$

H is the usual quantum mechanical Hamiltonian and is usually ignored in collapse theories, except when trying to find specialized experimental tests of collapse. (This equation of motion gives a non-unitary time evolution.) Space is divided up into fixed cubes, labeled by *n*, of size on the order of $\alpha^3 = (10^{-5})^3$ cm^3. The η_n are number operators; that is, when operating on a given version of reality, they give the number of particles in the n^{th} cube. λ is a frequency on the order of 10^{-16}/sec, and $w_n(t)$ is the random part of the potential energy at time t acting on the particles in the n^{th} volume element. $w_n(t)$ is related in the Pearle formulation to a random walk (or Brownian motion) parameter, $B_n(t)$ by

$$w_n(t) = \frac{dB_n(t)}{dt} \tag{2}$$

Let us now suppose the state vector is the sum of *K* potential versions of reality,

$$|\Psi,0\rangle = \sum_{k=1}^{K} a(k)|k\rangle \tag{3}$$

Then if we set *H*=0, Eq. (1) (for *given* $w_n(t)$) can be solved. Its solution is of the form

$$|\Psi,\mathrm{t}\rangle = \sum_{k=1}^{K} a(k)\beta_k(B_1(t),\dots,B_N(t))|k\rangle = \sum_{k=1}^{K} \alpha(\mathrm{k},\mathrm{t})|\mathrm{k}\rangle \tag{4}$$

where the β ‘s are exponential functions of the *B*’s, and there are *N* volume elements.

The probability for a given set of $w_n(t)$ in the Pearle model is presumed to be

$$\begin{aligned} P\big(w_1(t),\dots,w_N(t)\big) &= \left(\prod_{n=1}^{N} P_0(w_n(t))\right)\langle\Psi,\mathrm{t}|\Psi,\mathrm{t}\rangle \\ &= \left(\prod_{n=1}^{N} P_0(w_n(t))\right)\left(\sum_{k=1}^{K} |\alpha(\mathrm{k},\mathrm{t})|^2\right) \end{aligned} \tag{5}$$

where $P_0(w_n(t))$ is a white noise function. Roughly, Eq. (5) says that those random walks which increase the norm are favored (the norm does not stay constant in this model as it does in conventional linear, unitary quantum mechanics). Note that this equation does not hold at just one time; it holds at each instant (so the *B*'s are forced to 'walk' in time towards their more probable values).

Note also that this is a *nonlinear* model because Eq. (5) implies that the $w_n(t)$ depend (probabilistically) on the coefficients $\alpha(\mathrm{k}, \mathrm{t})$, so that $w_n(t) = w_n(t, \alpha(\mathrm{k}, \mathrm{t}))$. A different set of starting *a*'s will yield a different set of functions $w_n(t)$. Thus the Hamiltonian, through its dependence on the *w*'s, depends on the coefficients, so that Eqs. (4) and (5) together imply this is not a linear model.

> (One can see the nonlinearity more clearly if one imagines integrating the Hamiltonian equation on a computer, step by small time step. The choice of *w*'s or *B*'s at each step depends on the α's at that step, and the α-dependent choice of *w*'s is put back into the Hamiltonian equation.)

It is Eq. (5), with its dependence on the values of the *a*'s *in all the different versions*, which makes this model radically different from current quantum mechanics.

Difficulties with the Model.

There is nothing mathematically wrong with the model, but it has several questionable physics-related features which make it unlikely, in my opinion, that it is a 'correct' description of collapse. These are given in Ch. 19.

Problem with particle number as hook.

The GRWP approach uses particle number as the 'hook' upon which to hang state vector reduction. If this is to work, then there must be regions in the different versions of reality which have a sufficiently large difference in particle number. The simplest example is to have a pointer which points in different directions for different outcomes. Then the cubes where the pointer is located will be filled for a particular outcome, but empty for other outcomes. We will argue that there are situations where all regions have essentially the same number of particles for almost all outcomes. Thus this 'hook' is insufficient.

To start, we note that, as the GRWP collapse process proceeds, the ratio of the size of the collapsing part of the state to the non-collapsing part is approximately

$$e^{-\lambda \mathrm{t}(\Delta N)^2} \tag{6}$$

so the approximate time for collapse is, say,

$$\lambda \mathrm{t}(\Delta N)^2 \approx 5 \text{ or } t \approx 5/(\lambda(\Delta N)^2)$$

(7)

As we said before, $\lambda \approx 10^{-16}$/sec. ΔN is the change in the number of particles in some particular volume, fixed in space, between the non-detecting and the detecting state. The size of the volume is taken to be $\alpha^3 = (10^{-5})^3$ cm^3. In the case of a pointer, ΔN is the number of particles in an appropriately located 10^{-15} cm^3 cube (the cube is filled with particles in one position and empty in the other), which is approximately 3×10^{10} for normal densities. If we put these numbers into Eq. (7), we have a time of collapse of approximately 5×10^{-5} sec, which is adequate (any collapse time less than 10^{-3} sec is physiologically undetectable).

But this situation—where the cube is empty in, say, the non-detecting position and filled with particles in the position corresponding to detection—is a best-case scenario. There are situations where there is very little, if any, difference in the number of particles in any cube between the non-detecting and the detecting state:

> What if the readout device has a liquid crystal display? In any given 10^{-15} cm^3 cube, there is virtually no change in the number of particles between the state where it reads 0 and the state where it reads 1.
>
> What if the detection/readout device is a grain of film? Again, there is virtually no change in the number of particles (in a given cube) between the unexposed and the exposed states.

In these cases, because $\Delta N \ll 10^{10}$, the collapse time is unacceptably long. One possible way out is to adjust the values of α and λ, but the reduction in ΔN is so large in these and probably other cases that this almost certainly won't work.

A second possibility is to use a different 'hook' (property) to trigger reduction. But it seems difficult to come up with a hook that works in all situations. (Use of the mass in a cube is subject to the same criticism as the use of particle number.) So the 'hook' difficulty seems to me to be a serious problem for the GRWP collapse scheme.

Note on bubble chambers and slow-motion collapse.

Suppose we use a bubble chamber as a detector and operate it very near the boiling point. Then a particle going though it will nucleate bubbles in slow motion. That means the ΔN will grow slowly, and so the collapse will occur in slow motion. It would seem one could investigate collapse in this way, but I have not been able to devise a means of doing so.

References.

[1] Philip Pearle, *How stands collapse I*, arXiv, quant-ph/0611211v1 (2006).
[2] Philip Pearle, *How stands collapse II*, arXiv, quant-ph/0611212v3 (2007).
[3] G. C. Ghirardi, A. Rimini and T. Weber, *Unified dynamics for microscopic and macroscopic systems*, Phys. Rev. D**34**, 470 (1986).