## A18.1 A linear, non-unitary time translation operator cannot yield the probability law.

In Ch. 18, we showed that standard quantum mechanics—in which the time translation operator T(t) is linear and unitary—could not account for the probability law. The reason is that there is no singling out of just one version on each run. One might conjecture that the linearity could be kept and at the same time one could have singling out if *T*(*t*) is non-unitary and causes a *collapse* to just one version of reality. But we find that cannot be (because there must be coordination between branches, and that is not possible in a linear theory). Therefore the probability law implies that mathematical collapse, if it exists, must be non-linear.

**Non-unitary time translation.**

If the time translation is unitary, as it is in conventional theory, then the norms of the different branches of the wave function stay the same forever. Thus there can be no collapse to just one version. The question is what happens if we allow time translation to be non-unitary. Suppose in particular that some part of the Hamiltonian can change only the *norm* of the wave function, with the change, and therefore the linear time translation operator, depending on the random variables *w*. Then we have

$$\begin{aligned} |\,\Psi(\mathrm{t,w})\,\rangle &= T(t,w)|\,\Psi(0)\,\rangle \\ &= T(t,w)[\textstyle\sum a_k|\,\psi_k\rangle] \\ &= \sum a_k T(t,w)|\,\psi_k\rangle \\ &= \sum a(k)\beta_k(t,w)\,|\psi_k\rangle \end{aligned} \qquad (1)$$

where we have ignored the non-collapse part of the time evolution. (All $|\beta|$s are 1 if *T* is unitary.) In this case, it might happen that after some time has elapsed, for some value of *i*,

$$\begin{aligned} \frac{\beta_i^2(t,w)}{\sum_k \beta_k^2(t,w)} &\to 1, \qquad t \to \infty \\ \frac{\beta_j^2(t,w)}{\sum_k \beta_k^2(t,w)} &\to 0, \quad j \neq i \end{aligned} \qquad (2)$$

so we end up with a system which has collapsed to state *i*. Thus linearity does not rule out simple collapse (no probability law) if the time evolution is non-unitary.

**The $|\,a(i)\,|^2$ probability law requires collapse to be nonlinear.**

But now we must see whether the conjectured collapse of Eq. (2) can be made consistent with the probability law. We do N runs of the experiment (N>>1), with subscript *m* denoting a particular run. Then to satisfy the probability law, and taking into account Eq. (2), we must have for each *i*,

$$\frac{1}{N}\sum_{m}\frac{\beta_{i,m}^{2}(t,w_m)}{\sum_k \beta_{k,m}^{2}(t,w_m)} = |a(i)|^2, \qquad t, N \to \infty \tag{3}$$

But because the operator $T(t,w)$ is linear, its effect on the ket $|\psi_k\rangle$ must be independent of the coefficients. Thus we see from the last two lines in Eq. (1) that $\beta_{j,m}(t,w_m)$ must be independent of the *a*s. This implies the LHS of Eq. (3) is independent of the *a*s while the RHS is not, and so the equation cannot hold.

We have therefore shown that a linear, non-unitary time translation operator cannot lead to the probability law.