A16.2. The double-slit quantum eraser experiment. [1]

Simple double-slit.

In this experiment, the double-slit experiment is done under various circumstances to show how peculiar the implications of quantum mechanics are.

Case 1: Simple double-slit

We start with the simple double slit experiment of Ch. 5. After the slits, we can write the wave function as ′

$$|\Psi\rangle_1 = \int dz\,[\psi_{s1}(z)|z\rangle + \psi_{s2}(z)|z\rangle]/\sqrt{2} \tag{1}$$

where z is the distance measured along the detecting screen . The two waves, from slits s1 and s2, essentially differ by just a phase factor so that

$$\psi_{s2}(r) = e^{i\delta}\psi_{s1}(r) \tag{2}$$

$$\delta = \frac{2\pi d sin(\theta)}{\lambda} = \frac{2\pi d}{\lambda L} z = kz \tag{3}$$

where λ is the wavelength, d is the distance between slits, and L is the distance to the screen. Thus because of Eq. (2), the coefficients from the two slits add in $|\Psi\rangle_1$ so it becomes

$$|\Psi\rangle_1 = \int dz\,(1 + e^{i\delta})\psi_{s1}(z)|z\rangle/\sqrt{2} \tag{4}$$

From the probability law, the probability of perceiving an exposed grain at a distance z from the center of the detecting screen is proportional to the magnitude squared of the coefficient on $|z\rangle$ and is therefore proportional to

$$|\psi_{s1}(z)|^2|1 + e^{ikd}|^2/4 = |\psi_{s1}(z)|^2(1 + \cos(\delta))/2 \tag{5}$$

The $|\psi_{s1}(z)|^2$ term comes from the superimposed single-slit pattern. It is the same as the S(z) of (A5.1); $S(z) = (1 - \cos(k' z))/k'^2, k' = 2\pi w/\lambda L$ where w is the width of the slit. Thus the probability of observing an exposed grain at z is

$$P(z) \propto (1 + \cos(\delta))S(z)/2 \tag{6}$$

Case 2: Entangled photons, linear polarization.

We now suppose that each photon sent through the slit is one of a pair of entangled photons, with the photons having opposite polarizations. Following the notation of Walborn, et al [1], the photon that goes through a slit is labeled by subscript s (or s1 or s2) and the other photon is labeled by subscript p

The two linear polarization states are labeled by x and y, and the full two-photon state at the screen is

$$|\Psi\rangle_2 = [|\psi\rangle_1 + |\psi\rangle_2]/\sqrt{2} \tag{7}$$

$$|\psi\rangle_j = \int dz\, \psi_{sj}(z)\{|z,x\rangle_s|y\rangle_p + |z,y\rangle_s|x\rangle_p\}/\sqrt{2},\ j = 1{,}2 \tag{8}$$

with

$$\psi_{s2}(r_m) = e^{i\delta}\psi_{s1}(r_m) \tag{9}$$

Thus the full wave function is

$$|\Psi\rangle_2 = \int dz\, \psi_{s1}(r_m)(1 + e^{i\delta})\{|z,x\rangle_s|y\rangle_p + |z,y\rangle_s|x\rangle_p\}/2, \tag{10}$$
$$= \int dz\ \ |\Psi_2(\mathrm{z})\rangle \tag{11}$$

The probability of observing a grain exposed at z is proportional to

$$\langle\Psi_2(z)|\Psi_2(z)\rangle. \tag{12}$$

The x and y polarization states, as well as the s and p states are orthogonal, and so we get the same probability as in Eq. (6). That is, there is still the usual interference.

Case 3: Entangled photons, circular polarization.

Next we put a quarter wave plate in front of each slit. (It must have been very difficult to do this because the two slits are very close together.) These are arranged so that on slit 1, this changes linear polarization state $|x\rangle$ to circular polarization state $|L\rangle$ and linear polarization state $|y\rangle$ to circular polarization state $i|R\rangle$, while on slit 2 this changes linear polarization state $|x\rangle$ to circular polarization state $|R\rangle$ and linear polarization state $|y\rangle$ to circular polarization state $-i|L\rangle$, with $\langle \mathrm{L}|\mathrm{R}\rangle = 0$. Thus we have 'marked' the wave functions from the two different slits and do not expect a double-slit interference pattern.

To see that this does indeed happen, we proceed as before. The full wave function is

$$|\Psi\rangle_3 = [|\psi\rangle_1 + |\psi\rangle_2]/\sqrt{2} \tag{13}$$

$$|\psi\rangle_1 = \int dz \psi_{s1}(z)\left\{|z,L\rangle_s|y\rangle_p + i|z,R\rangle_s|x\rangle_p\right\}/\sqrt{2} \tag{14}$$

$$|\psi\rangle_2 = \int dz \psi_{s2}(z)\left\{|z,R\rangle_s|y\rangle_p - i|z,RL|x\rangle_p\right\}/\sqrt{2} \tag{15}$$

Using equations similar to (11) and (12), as well as the orthogonality properties, we see that we indeed get no interference. The states that go through the two slits have been 'marked' so they cannot interfere.

Case 4: Quantum 'erasure' of the single-slit pattern.

We obtained a single-slit pattern in case 3 because we distinguished the wave function that went through s1 from the one that went through s2. But strangely, by playing with the p photons (which don't go through the slits!), one can resurrect the double-slit interference pattern for the s photons. That is, we can 'erase' the single-slit pattern.

To see how this comes about, we first change bases for the linearly polarized photons. Let

$$
\begin{aligned}
|x\rangle_p &= \cos(\beta)|x'\rangle_p + \sin(\beta)|y'\rangle_p \\
|y\rangle_p &= \cos(\beta)|y'\rangle_p - \sin(\beta)|x'\rangle_p
\end{aligned} \tag{16}
$$

The full state vector then consists of the sum of four terms for each slit. For s1, they are

$$
\begin{aligned}
|\psi\rangle_{1a} &= \int dz\ \psi_{s1}(z)\{\,|\psi(L,z)\rangle[\cos(\beta)]\ |y'\rangle_p\ |\}/\sqrt{2} \\
|\psi\rangle_{1b} &= \int dz\ \psi_{s1}(z)\{\,|\psi(L,z)\rangle\left[-\sin(\beta)|x'\rangle_p\right]|\}/\sqrt{2} \\
|\psi\rangle_{1c} &= \int dz\ \psi_{s1}(z)\{\,i|\psi(R,z)\rangle\left[\cos(\beta)\,|x'\rangle_p\right]\}/\sqrt{2} \\
|\psi\rangle_{1d} &= \int dz\ \psi_{s1}(z)\{\,i|\psi(R,z)\rangle\,[sin(\beta)|y'\rangle_p]\,\}\ \ /\sqrt{2}
\end{aligned} \tag{17}
$$

And for s2 they are

$$
\begin{aligned}
|\psi_{2a} &= \int dz\ e^{i\delta}\psi_{s1}(z)\{\,|\psi(R,z)\rangle\left[\cos(\beta)\,|y'\rangle_p\right]|\}/\sqrt{2} \\
|\psi\rangle_{2b} &= \int dz\ e^{i\delta}\psi_{s1}(z)\{\,|\psi(R,z)\rangle\left[-\sin(\beta)|x'\rangle_p\right]|\}/\sqrt{2} \\
|\psi\rangle_{2c} &= \int dz\ e^{i\delta}\psi_{s1}(z)\{-\,i|\psi(L,z)\rangle\left[\cos(\beta)\,|x'\rangle_p\right]\}/\sqrt{2} \\
|\psi\rangle_{2d} &= \int dz\ e^{i\delta}\psi_{s1}(z)\{-\,i|\psi(L,z)\rangle\,[sin(\beta)|y'\rangle_p]\,\}\ \ /\sqrt{2}
\end{aligned} \tag{18}
$$

These get added together to give $|\Psi\rangle_4$

$$
\begin{aligned}
\sqrt{2}|\Psi(\mathrm{z})\rangle_4 = &|\psi(R,z)\rangle|y'\rangle_p[i\sin(\beta) + e^{i\delta}\cos(\beta)] + \\
&|\psi(L,z)\rangle|y'\rangle_p\left[\cos(\beta) - ie^{i\delta}\sin(\beta)\right]\} + \\
&|\psi(R,z)\rangle|x'\rangle_p[i\cos(\beta) - e^{i\delta}\sin(\beta)] \\
&|\psi(L,z)\rangle|x'\rangle_p[-\sin(\beta) - ie^{i\delta}\cos(\beta)]
\end{aligned} \tag{19}
$$

The four states are orthogonal, so when we calculate $\langle\Psi_4(z)|\Psi_4(z)\rangle$, we add together the squares of all four terms and get no interference; that is, we get, the single-slit pattern.

But now comes the 'erasure.' Suppose we put a polarizer in the path of the p photon—not the s photon that goes through the slits!—so that only $|x'\rangle_p$ photons are let through. And suppose we only count those s photons which give a coincidence with the p photons (so the detected s and p photons are parts of an entangled pair). Then $|\Psi_4(z)\rangle$ will only consist of the last two terms. In that case, we do indeed get interference,

$$\langle\Psi_4(z)|\Psi_4(z)\rangle_{x'} \propto |\psi_{s1}(z)|^2 S(z)[1 - \sin(2\beta)\sin(kz)]/2 \tag{20}$$

One finds that the experimental results [1] do indeed follow this quantum mechanically derived result. And so, without tinkering with the s photon (we can detect the p photon long after the s photon has been detected), we have ‘erased’ the single-slit pattern and arrived at a pattern with interference!

How are we to explain this sleight-of-hand? To see, we also give the probability when the coincidence counting uses the y' photon rather than the x' photon;

$$\langle\Psi_4(z)|\Psi_4(z)\rangle_{y'} \propto |\psi_{s1}(z)|^2 S(z)[1 + \sin(2\beta)\sin(kz)]/2 \tag{21}$$

We see that the two interference patterns just add up to the (interference-free) single-slit diffraction pattern! So what is happening is that the **coincidence counting** sorts the photons into two piles, one for each polarization state of the p photon. Thus the detection of the p photon in one polarization state or the other is not really influencing (after the fact) where the s photon lands; instead it is only sorting out which s photons go with which p polarization states.

The interest of the experiment lies in how difficult it is to explain the results if one supposes there are particulate photons. How could a measurement on the p (particulate) photon—after the s photon has been detected—affect where the s (particulate) photon hits the screen?

References.

[1] S. P. Walborn,1 M. O. Terra Cunha,1,2 S. Pa´dua,1 and C. H. Monken Double-slit quantum eraser, Phys. Rev. A, 033818 (2002)