A13.1. Technicalities of parts of a wave function.

In classical physics, a small part of a wave, a water wave for example, carries only a correspondingly small part of the energy of the wave. But this statement is misleading in quantum mechanics. To see this, we again use the half-silvered mirror experiment (Ch. 6). The linear energy operator $\mathcal{E}$, applied to the wave function after the mirror, gives

$$\begin{aligned}&\mathcal{E}\,[|ph\rangle]=\\&\mathcal{E}\,[a(H)|ph\ H\rangle+a(V)|ph\ V\rangle]=\\&a(H)\,\mathcal{E}\,[|ph\ H\rangle]+a(V)\,\mathcal{E}\,[|ph\ V\rangle]\end{aligned} \tag{1}$$

with "ph" standing for photon. If we now assume the initial photon state, $|ph\rangle$, is an eigenstate of the energy with eigenvalue E, and that energy is conserved, then

$$E[a(H)|ph,\ H\rangle+a(V)|ph,\ V\rangle]=a(H)\,\mathcal{E}\,|ph\ H\rangle+a(V)\,\mathcal{E}\,|ph\ V\rangle \tag{2}$$

We set $a(H)=\exp(i\theta)|a(H)|$, $a(V)=|a(V)|$ and note that $|ph,\ H\rangle$, $\mathcal{E}\,|ph,\ H\rangle$, $|ph,\ V\rangle$, $\mathcal{E}\,|ph,\ V\rangle$ and E are all independent of θ. Then taking the derivative of Eq. (2) with respect to θ shows that

$$\mathcal{E}\,|ph,\ H\rangle=E|ph,\ H\rangle \tag{3}$$

and similarly for $|ph,\ V\rangle$. That is, the two kets after the mirror are separately eigenstates of the energy, with the same eigenvalue as the original state. (Note, however, that if the experiment is repeated many times, the horizontal branch will carry an *average* energy of $|a(H)|^2E$ because of the probability law.) This property—that each branch carries the full energy (and momentum)—is necessary for the Compton and photoelectric effects of Ch. 13, and also for Ch. 14, where a small part of the wave function has the energy necessary to expose a film grain.