A8.1 Unitarity.

Unitarity is a technical property of some linear operators, the time translation operator—the operator that takes a state vector from one time to another—in particular. It says that the norm of a state is not changed by the operator. This is equivalent to saying that the hermitian adjoint of the operator, $U^\dagger$, is equal to its inverse, U^{-1}. If $U(t)$ is the time translation operator, then, with H being the hermitian Hamiltonian,

$$U(t) = e^{iHt}, H^\dagger = H \qquad (1)$$
$$U^\dagger(t) = e^{-iHt} = U^{-1}(t)$$

To see that the norm (A1.1) stays the same under time translation (or a unitary operator in general), note that the norm squared of $U(t)|\,\psi\rangle$ is

$$(U(t)|\,\psi\rangle)^\dagger(U(t)|\,\psi\rangle) = \langle\psi|U^\dagger(t)U(t)|\,\psi\rangle = \langle\psi|U^{-1}(t)U(t)|\,\psi\rangle = \langle\psi|\,\psi\rangle \qquad (2)$$

If $|\psi\rangle = \sum a(i)|i\rangle$, with the basis vectors orthonormal, then $\langle\psi|\,\psi\rangle = \sum|a(i)|^2$. See also A1.1.