A6.2 Kets, state vectors, and wave functions.

Notational device.

Kets were introduced by Dirac as a way of denoting states when the mathematics is linear (A1.1). So they are primarily a notational device (although they conceal a secret, the fourth mystery!).

Examples.

In Ch. 5 and A5.3, kets are used to denote the polarization states of a photon |x⟩ or |y⟩. In Ch. 6, they are used to denote the paths taken by 'parts' of a photon,

$$
\begin{aligned}
|\text{state of single photon}\rangle = &|\text{part 1 of the wave function on the H path}\rangle \\
&\oplus \\
&|\text{part 2 of the wave function on the V path}\rangle
\end{aligned} \quad (1)
$$

In A6.1, they are used to denote the space and spin states of a spin 1 particle, (Note: A not-fully-consistent notation is used in this state: $\psi_{+1}(x)$ should really be $\int d^3x\psi_{+1}(x)|\mathrm{x}\rangle$) and similarly for $\psi_0(x)$ and $\psi_{-1}(x)$.)

$$|\text{after}\rangle = a(+1)|+1\rangle\psi_{+1}(x) + a(0)|0\rangle\psi_0(x) + a(-1)|-1\rangle\psi_{-1}(x) \quad (2)$$

as well as the states of versions of detectors and observers;

$$
\begin{aligned}
|\text{after}\rangle = &a(+1)|+1\rangle|D(+),\text{yes}\rangle|D(0),\text{no}\rangle|D(-),\text{no}\rangle|\text{Obs: yes,no,no}\rangle + \\
&a(0)|0\rangle|D(+),\text{no}\rangle|D(0),\text{yes}\rangle|D(-),\text{no}\rangle|\text{Obs: no,yes,no}\rangle + \\
&a(-1)|-1\rangle|D(+),\text{no}\rangle|D(0),\text{no}\rangle|D(-),\text{yes}\rangle|\text{Obs: no,no,yes}\rangle
\end{aligned} \quad (3)
$$

They are also used in Ch. 12 to denote the state of a 'particle;'

$$|\psi\rangle = |m, E, p, S, s_z, Q\rangle \quad (4)$$

But as we argue in Part II, it is a virtual certainty there are no particles. So we don't really know **what the kets stand for** in quantum mechanics. All we can say at present is that they stand for the state of whatever it is that makes up matter. (This is pursued in Part IV.)

The state vector.

We have implied in various chapters of Part I that the state of a 'particle' or system is exemplified by its wave function. But it is more correct to say that it is properly described by its state vector. In Eq. (2) for example, the state vector is a sum of kets (with each ket multiplied by a coefficient). We have often used the wave function language rather than the state vector language because the wave function can be visualized (as a mist) but there is no such visualization for the state vectors. The state vector language is actually more correct, but for many explanatory purposes, there is very little distinction.

Relation between the state vector and the wave function.

There are kets, $|x\rangle$, that denote a 'particle' at position x. The state vector is then

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$$|\Psi\rangle = \int d^3x\, \psi(x,t)|x\rangle \tag{5}$$

where $\psi(x,t)$ is the wave function at time t. It tells 'how much' of 'the particle' is located at x. Or rather, $|\psi(x,t)|^2$ gives the probability of finding 'the particle' at x. (But this language is all wrong because there is no evidence for particles.)

When one has discrete set of states $|i\rangle$, we write that state vector as $|\psi\rangle = \sum a(i)|i\rangle$ with the condition $\sum |a(i)|^2 = 1$. And when one has a continuous set of states $|x\rangle$, the x takes the place of the *i* and we write the state as in Eq. (5). In place of $\sum |a(i)|^2 = 1$, we then have $\int d^3x |\psi(x)|^2 = 1$. From these equations, and remembering that an integral is really just a sum, we see that the wave function $\psi(x)$ is the same type of object as the coefficients *a*(*i*)except that the 'index' *x* (in place of *i*) can take on a continuum of values.