A6.1. Spin and the Stern-Gerlach Experiment.

Discrete values of angular momentum, or spin as it is called on the atomic level, are one of the hallmarks of quantum mechanics. And the Stern-Gerlach experiment, used to measure spin on the atomic level, is often used to illustrate ideas in quantum mechanics. It can be used to demonstrate the quantization of angular momentum, how abstract mathematics yields physically relevant results, the idea of several simultaneously existing versions of reality (Ch. 6), and one aspect of the particle-*like* behavior of the wave function.

The Stern-Gerlach experiment.

The set-up for the Stern-Gerlach experiment is as shown in Fig. (A6.1-1).

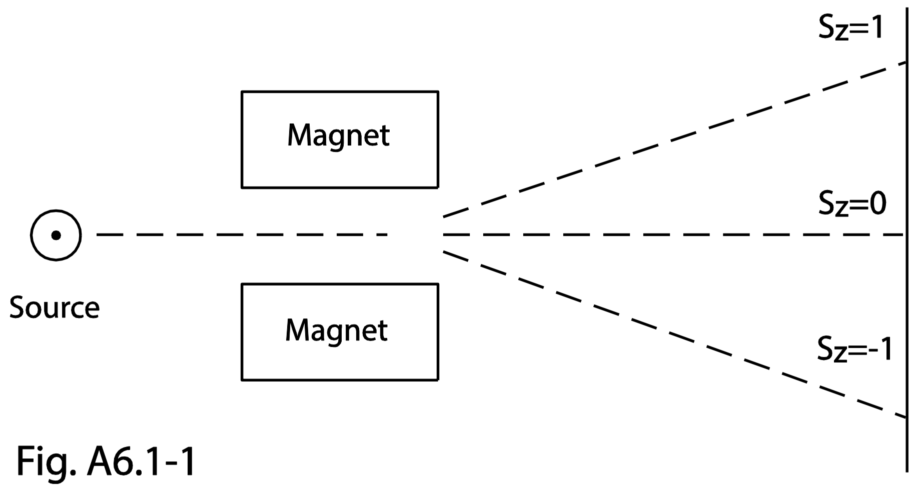

Fig. A6.1-1

Figure A6.1-1. The Stern-Gerlach experiment for measuring the spin of an atomic-level system. The states of the wave function with different z component of spin, s_z, travel in slightly different directions after passing through the magnetic field. Note that this diagram is deceptive, just as Fig. (6-1) of Ch. 6 is deceptive.

A particle—particle-like wave function—moving in the x-direction passes through a magnetic field. The wave function is a linear combination of states having different s_z and the magnetic field exerts different forces on the different states. Because of this, the states of the wave function with different spins exit the magnetic field traveling in slightly different directions. Suppose, for example, that a spin 1 particle is shot into the magnetic field. Then three different possible trajectories for the wave function will emerge from the apparatus; the +1 spin might be traveling slightly upward in the z direction, the spin 0 would travel straight through, and the –1 spin would be travelling slightly downward, as shown in the figure.

Several versions of reality. Particle-like behavior

Suppose the wave function for the particle before it reaches the magnetic field is a linear combination of the three possible spins,

$$|\text{before}\rangle = a(+1)|+1\rangle\psi(x) + a(0)|0\rangle\psi(x) + a(-1)|-1\rangle\psi(x) \quad (1)$$

where the kets, $|\ldots\rangle$ indicate the spin part of the wave function, the a's are constant coefficients (Ch. 8) and ψ(x) represents the spatial part of the wave function (presumed to be the same for all three before going through the magnet). After the particle goes through the magnetic field, the wave function becomes

$$|\text{after}\rangle = a(+1)|+1\rangle\psi_{+1}(x) + a(0)|0\rangle\psi_0(x) + a(-1)|-1\rangle\psi_{-1}(x) \quad (2)$$

where the subscripts indicate that the three different spatial parts of the wave function take three different paths.

We now put detectors, D(+), D(0), D(–), on each of the paths and suppose there is an observer looking at the outcome (*yes* or *no*) displayed on each dial. After detection and observation, the full wave function, particle plus detectors plus observer, is then the sum of three parts;

$$\begin{aligned}|\text{after}\rangle = {} & a(+1)|+1\rangle|D(+),\text{yes}\rangle|D(0),\text{no}\rangle|D(-),\text{no}\rangle|\text{Obs: yes,no,no}\rangle + \\ & a(0)|0\rangle|D(+),\text{no}\rangle|D(0),\text{yes}\rangle|D(-),\text{no}\rangle|\text{Obs: no,yes,no}\rangle + \\ & a(-1)|-1\rangle|D(+),\text{no}\rangle|D(0),\text{no}\rangle|D(-),\text{yes}\rangle|\text{Obs: no,no,yes}\rangle\end{aligned} \quad (3)$$

We see that the final state contains three separate versions of reality. In particular, there are three versions of the observer, with each perceiving a different result. One and only one of the versions will correspond to what we, as 'the' observer, perceive, but quantum mechanics does not say which one. (It does say, however, that if the experiment is repeated many times, the +1 result will be observed on a fraction $|a(+1)|^2$ of the runs, and similarly for 0 and -1.) Thus the diagrammatic representation of Eq. (3), analogous to Fig. (6-3) of Ch. 6, is Fig. (6-2). One and only one detector is activated on each run, exactly as if there was a particle traveling on just one of the trajectories.

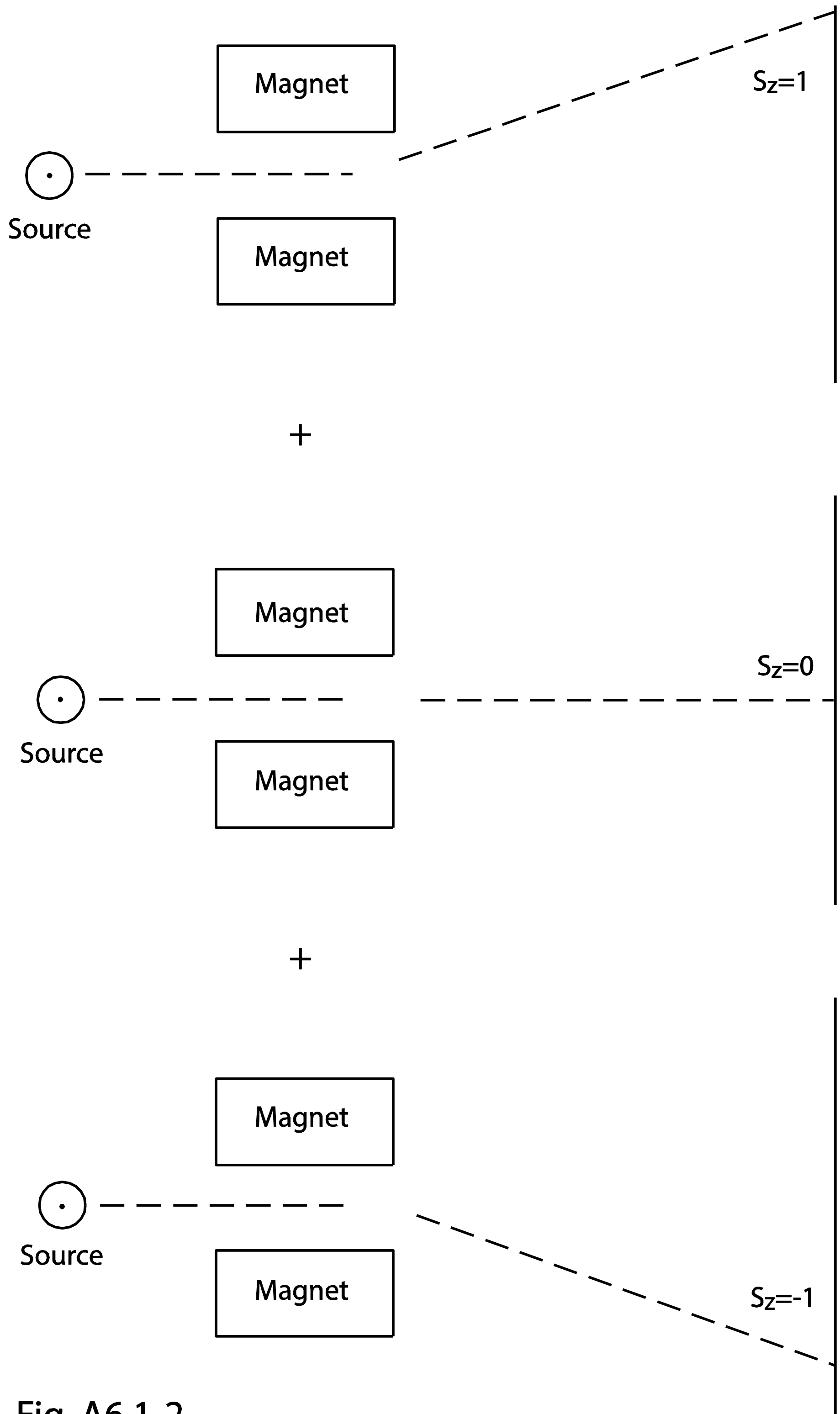

Fig. A6.1-2

Figure 2. Three possible outcomes to the spin experiment. The net perceived effect from the quantum mechanical mathematics is as if there were a particle on just one of the trajectories, even though there is no particle in the mathematics.

Classical and quantum angular momentum.

Angular momentum was defined in classical physics centuries ago; it is the 'momentum around the center' (momentum times the radius) a 'particle' has when traveling in a circle. This could take on any value so that classically, angular momentum is a continuous variable.

But that is not true in quantum mechanics. Instead, when measured along a given direction, angular momentum (spin) can only take on certain discrete values. For a spin 1 wave function, the allowed values (in units of the Planck constant $\hbar$) are 1,0,–1. For a spin 2 wave function, they are 2,1,0, –1, –2, and so on. This *quantization* (only a discrete number of values allowed, instead of a continuum) of angular momentum is one of the primary characteristics of quantum mechanics. It is derived in A12.3.

Spin ½ .

There is one more intriguing point about angular momentum in quantum mechanics. For the hydrogen atom, the mathematics predicts that the *total* angular momentum (in the proper units) can be 0, 1, 2, 3, 4,… and so on, with, respectively, 1, 3, 5, 7, 9… *components* when measured along some axis. That is, there are an odd number of components (different states, different number of trajectories after the magnetic field). But when Stern and Gerlach measured the angular momentum of silver atoms, they found only *two* components, corresponding to a spin of ½ (with allowed values + ½ and – ½)! This, in fact, was how spin ½ was discovered.

Interestingly, the abstract group representational theory (with 'abstract' here meaning not associated with a particular problem) permits the observed spin ½, even though it does not occur in the hydrogen atom case (see Ch. 12 and A12.3). And in fact, many of the 'elementary particles'—including electrons, neutrinos, quarks, protons, and neutrons—have spin ½.