A5.1 Interference.

Simple interference.
Interference is the quintessential wave property. It occurs when two waves of the same type meet and (usually) travel in nearly the same direction. To give the simplest example, suppose we have two waves of equal amplitude

$$\psi_1(x,t) = e^{i\omega t - ikx}/\sqrt{2}$$
$$\psi_2(x,t) = e^{i\varphi}e^{i\omega t - ikx}/\sqrt{2} \qquad (1)$$

both traveling in the *x* direction, with

$$\omega = 2\pi f = 2\pi c/\lambda, \;\; k = 2\pi/\lambda \qquad (2)$$

f is the frequency, λ is the wavelength, *c* is the velocity of the wave, and φ is the phase angle between the two waves. The rule for adding these two waves is simply the usual algebraic addition, so the resulting wave is

$$\psi_r(x,t) = \psi_1(x,t) + \psi_2(x,t) = \left(1 + e^{i\varphi}\right)e^{i\omega t - ikx}/\sqrt{2}$$
$$= \sqrt{(1+\cos(\varphi))/2}\, e^{i\beta}e^{i\omega t - ikx}, \qquad (3)$$
$$\tan(\beta) = \sin(\varphi)\,/(1+\cos(\varphi)\,)$$

Thus the amplitude of the wave, $\sqrt{(1+\cos(\varphi))/2}$, ranges from 1 to 0, depending on the value of φ. If $\varphi = 0$ so the waves are completely in step, the amplitude is 1 and we have constructive interference; and if $\varphi = 180^o$ so the two waves are exactly out of step, the amplitude is 0 and we have destructive interference. If the amplitudes of the two original waves are unequal, one cannot have complete destructive interference.

Single slit.
We have a single slit of width s, as shown in Fig. (A5.1-1).

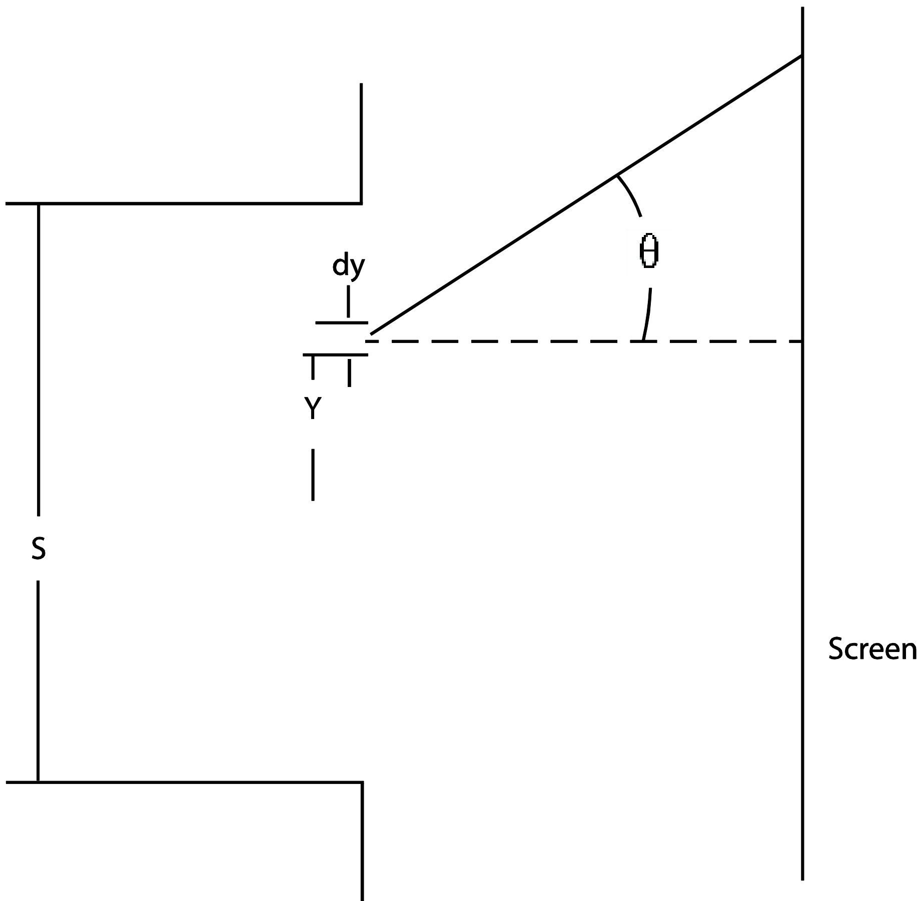

Fig. A5.1-1

Figure A5.1-1. The single slit. We consider the light from the small element dy emitted in the θ direction.

The vertical distance within the slit is denoted by y, measured up. At a distance y up, the wave from dy has traveled an extra distance (compared to the light from y=0) of ysin(θ). The extra phase is

$$\delta(y) = \frac{2\pi(y\, sin(\theta))}{\lambda} = ky,\ \ k = 2\pi sin(\theta)/\lambda \tag{4}$$

The net wave from the sum of the light from all the strips dy is

$$\int_0^w dy e^{iky} = \frac{e^{ikw}-1}{ik} \tag{5}$$

with w being the width of the slit. The magnitude squared of this as a function of θ is proportional to the absolute square of the expression in Eq. (5),

$$S(ks) = (1 - \cos(ks))/k^2 \tag{6}$$

This has a minimum at kz=2πn except at n=0. The graph of intensity is similar to that of Fig. 1 in Ch. 6, except the minima are further apart.

Double slit.

The two slits are a distance *d* apart, as shown in Fig. A5.1-2.

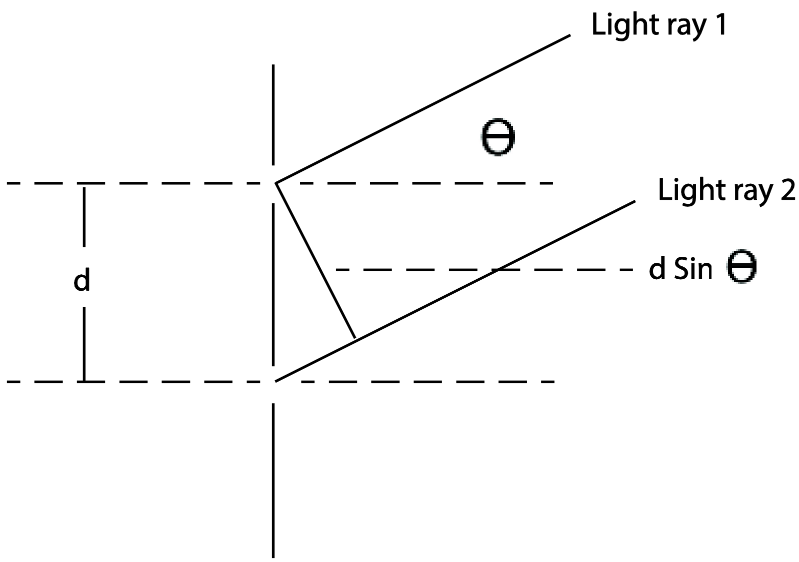

Fig. A5.1-2

Figure A5.1-2. The double slit. We consider light from the two slits emitted in the θ direction.

The difference in distance travelled by the two waves is $d\sin(\theta)$. And so the magnitude squared of the wave is

$$|1 + e^{ikd}|^2/2 = (1 + \sin(\delta)),\ \delta = 2\pi d\,\sin(\theta)/\lambda \qquad (7)$$
$$z = L\tan(\theta) \sim L\sin(\theta)$$

This has its first minimum at $d\sin(\theta) = dz/L = \lambda/2$.

We have left out the interference effects of each single slit. This will cause the result of Eq. (7) to be multiplied by the factor in Eq. (6).